

BOOK REVIEW

The Navier-Stokes Equations: A Classification of Flows and Exact Solutions.

By PHILIP DRAZIN & NORMAN RILEY. Cambridge University Press, 2006. 206 pp.
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As the authors comment, exact solutions of any partial differential system are worthy of attention but this is especially true of the subject of their book, the Navier–Stokes equations. There are many useful exact solutions which illuminate and inform our understanding of basic fluid mechanics and which provide a valuable platform for more intensive investigations and validation of computer codes. Although some exact solutions are mere artifacts of intellectual ingenuity, the authors' premise that it would be useful to provide an up-to-date, single-source compendium of all that has been documented in the literature on exact solutions of the Navier–Stokes equations is legitimate and laudable. In fact, the authors do not claim that the content is truly exhaustive. Their choice of material for inclusion in the book is however both comprehensive and judicious and references provide adequate access to outlying contributions to the literature sufficient for it to be considered an exhaustive source.

The introduction to the book outlines the scope of the text. After a succinct review of the derivation of the Navier–Stokes equations the authors clarify their understanding of what constitutes an exact solution of these equations. They wisely go beyond those that can be represented in terms of simple functions to include those that are expressed as solutions of ordinary differential systems, especially similarity solutions. Similarity solutions are of course manifestations of underlying invariances of the Navier–Stokes equations. Although originally derived from considerations of dynamic similarity, the authors pay tribute to the formal developments of Lie group theory and the associated uncovering of many new and interesting exact solutions that this has generated.

Beyond the introductory 'Scope' the authors choose to categorise exact solutions along temporal and geometric divides. Chapter 2 focuses on 'Steady flows bounded by plane boundaries' as generated by pressure gradients, sliding and stretching boundaries. Suction and injection effects are also included. Beltrami flows are introduced and indeed feature throughout the rest of the book. The usual suspects of Couette–Poiseuille flow and stagnation point flow necessarily feature in the early pages but more subtle and esoteric solutions quickly appear and issues of uniqueness command attention.

Chapter 3 is concerned with 'Steady axisymmetric and related flows'. Further to the natural extensions of Chapter 2, rotating flows including rotating disk flows and Ekman flow are featured. Exact solutions involving concentrated flows such as jets and vortices are also considered. Even more forcibly the richness and non-uniqueness of similarity solutions illuminate the discussion of a variety of configurations.

Time-dependent flows allowing for impulsive motion, oscillating pressure gradients and oscillating boundaries make up the subject matter of Chapter 4, 'Unsteady flows bounded by plane boundaries'. This includes a fulsome review of stagnation point flows under a range of constraints and an interesting section on squeeze flows. Finally

Chapter 5 explores 'Unsteady axisymmetric and related flows'. Natural extensions of Chapter 4 are complemented by the presentation of rotational flows and vortex motions.

An extensive set of references is witness to the thoroughness of the authors in seeking out material to complement their own wide-ranging familiarity and involvement in the subject matter.

I enjoyed reading this book and acknowledge the masterly way in which the material was presented in a uniformly consistent and coherent notation and style. The subject matter is logically sequenced and lucidly delivered in an insightful and authoritative commentary. Although not expressly presented as a textbook, the authors are right to suggest the value of the book as a source of examples and exercises that may help students in their voyage into fluid mechanics. It is certainly to be recommended as source of reference for applied mathematicians and fluid mechanicians who delight in elegance of solution, concision of presentation and comprehensive coverage of subject matter.

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